DETAILS EXPLANATIONS

Paper-1 (Paper-1)

[PART : A]

1. Maximum negative Bending Moment = $\frac{Wl}{8}$

$$BM = \frac{24 \times 10}{8} = 30 \text{ kN-m}$$

2.
$$\delta = \frac{5Wl^4}{384EI}$$

W = UDL - intensity

l = length

EI = Flexural-Rigidity

3. **Parallel Axis theorem :** Moment of Inertia about any axis parallel to centroidal axis is equal to the moment of Inertia about centroidal axis plus the Area multiplied by square of distance between the axis.

$$I = I_{CG} + A \cdot r^2$$

4.
$$I = \frac{BD^3}{3} = \frac{400 \times 200^3}{3} = 2.67 \times 10^8 \text{ mm}^4$$

5. **Principal Stresses :**

$$\sigma_1 \text{ or } \sigma_2 = \left(\frac{\sigma_{n_1} + \sigma_{n_2}}{2}\right) \pm \sqrt{\left(\frac{\sigma_{n_1} - \sigma_{n_2}}{2}\right)^2 + \tau_{xy}^2}$$

6. It is the temperature at which the creep is uncontrollable.

7.
$$E = 2G(1 + \mu)$$

$$G = \frac{2 \times 10^5}{2(1 + 0.28)}$$

$$G = 78125 \ \frac{N}{mm^2}$$

$\because$ For mild steel

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$\mu = 0.28$$

8.

$$M_X = -P.x$$

$$\text{Strain Energy (U)} = \int_0^L \frac{M_x^2 dx}{2EI}$$

$$U = \frac{P^2 L^3}{6EI}$$

9. Fixed end moment at 'A'.

$$M_{FAB} = \frac{10 \times 2 \times 3^2}{5^2} = 7.2 \text{ kN-m}$$

10. Factored load

Case (i) $= (35 + 25 + 12) \times 1.2 = 86.4$ kN-m

Case (ii) $= (35 + 25) \times 1.5 = 90$ kN-m

So, factored load = 90 kN-m

11. For controlling deflection in any beam the span to depth ratio is controlled.

12. Bending-stress 'f' $= \dfrac{M}{Z} = \dfrac{150 \times 10^6}{\left(\dfrac{100 \times 250^2}{6}\right)} = 144 \text{ N/mm}^2$

13. Coefficient of Uniformity

$$C_u = \frac{D_{60}}{D_{10}} = \frac{0.80}{0.28} = 2.85$$

Coefficient of Curvature

$$C_C = \frac{D_{30}^2}{D_{10} \cdot D_{60}} = \frac{0.34^2}{(0.28)(0.80)} = 0.516$$

14. If degree of consolidation U > 0.6

Time factor $T_V = -0.9332 - 0.0851 \log_{10}(1 - U)$

15. Effective Stress $(\bar{\sigma})$ = Total stress (σ_T) – Pore pressure (U)

$$\bar{\sigma} = \sigma_T - \gamma_w.h$$

$$\bar{\sigma} = 50 - (10 \times 2.5) = 25 \text{ kN/m}^2$$

16. $$V_V = V_S$$

$$\text{Void Ratio} = \frac{V_v}{V_s} = 1$$

$$\text{Porosity } n = \frac{e}{1+e} = \frac{1}{1+1} = 0.5$$

17. Gantry girders are used in workshops to transfer the load from one corner to another.

18. Stiffeners are used to prevent the buckling in girders in both horizontal and vertical direction.

19. For RC-Slabs minimum Reinforcement is

0.12% $\rightarrow$ For HYSD-bars

0.15% $\rightarrow$ For Mild-Steel

20. External indeterminacy is the number of support-reactions that can not be calculated using equilibrium equations only.

[PART : B]

21. $$\text{Poisson's Ratio} = \frac{1}{m} = \mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$= \frac{\left(\frac{4.5\times10^{-3}}{28}\right)}{\left(\frac{2}{1\times10^{3}}\right)} = \frac{4.5\times10^{-3}\times1\times10^{3}}{28\times2} = 0.08$$

22. $\because$ Strain Energy Ratio due to equal torsion (T)

$$\Rightarrow \frac{U_1}{U_2} = \left(\frac{d_2}{d_1}\right)^4 \Rightarrow \frac{40}{U_2} = \left(\frac{2d}{d}\right)^4$$

$$\Rightarrow U_2 = \frac{40}{16} = 2.5 \text{ kN-m}$$

23. The angle of twist (ϕ) at the point of application of torque will be same

$$\phi_1 = \phi_2$$

$$\frac{T_A L_A}{GI} = \frac{T_B L_B}{GI} \quad \text{...(i)}$$

Since Torsional Rigidity (G.I.) is same.

and $T = T_A + T_B$ ...(ii)

$$T_A(1) = T_B(3)$$

$$T_A = 3T_B$$

$$\Rightarrow \quad 3T_B + T_B = 600$$

$$T_B = 150 \text{ N-m}$$

$$\therefore \quad T_A = 600 - 150 = 450 \text{ N-m}$$

24. For Rectangular section :

$$q_{max} = \frac{3}{2}q_{avg}$$

Maximum Shear stress

$$(q)_{max} = 1.5 \times \frac{F}{B.D.}$$

$$(q)_{max} = 1.5 \times \frac{250\times10^3}{250\times350} = 4.28 \text{ N/mm}^2$$

25. If the other principal stress is tensile

$$\Rightarrow \quad \frac{P_1 - P_2}{2} = q$$

$$\Rightarrow \quad \frac{P_1 - 100}{2} = 100$$

$$\Rightarrow \quad P_1 = 300 \text{ MPa}$$

If the other principal stress is compressive

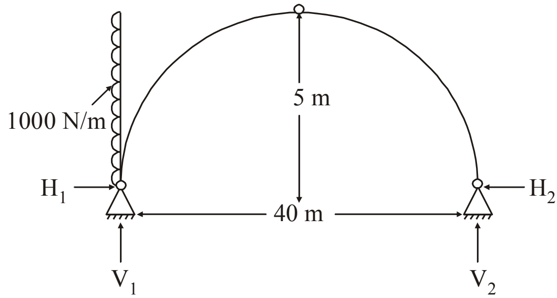
$$\frac{P_1 + 100}{2} = 100$$

$$\Rightarrow \quad P_1 = 100 \text{ N/mm}^2$$

26. For the vertical reaction at right support

$$V_2 = \frac{1000\times5\times2.5}{40}$$

$$V_2 = 312.5 \text{ N}$$



Considering right segment and taking moment about crown :

$$H_2 \times 5 = V_2 \times 20$$

$$\Rightarrow \quad H_2 = \frac{312.5 \times 20}{5} = 1250 \text{ N}$$

27. By the use of Betti's theorem,

Load at Z × 6 = (20 × 8) + (40 × 5)

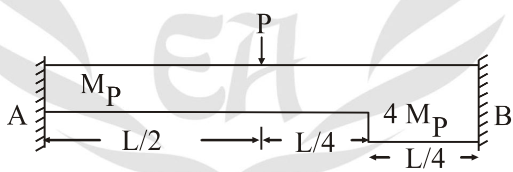
So, load at Z = $\left(\dfrac{160+200}{6}\right)$

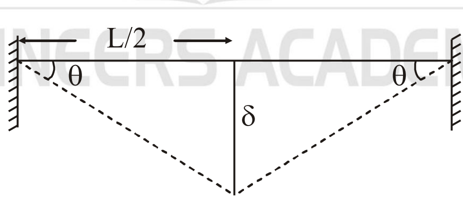
load at Z = 60 kN

28. By External work method

External work = Internal work

$$P\delta = M_P.\theta + M_P 2\theta + 4M_P\theta$$





$$\Rightarrow \quad P \cdot \frac{L}{2}\theta = 7M_P\theta$$

$$\Rightarrow \quad M_P = \frac{PL}{14}$$

$$\Rightarrow \quad M_P = \frac{20 \times 10}{14} = 14.29 \text{ kN-m}$$

29. Load Bearing Stiffners :

- Bearing stiffners are provided at the points of concentrated loads and at supports.
- Where these stiffners are to provide restraint against torsion of the plate girder at the ends.

$$I \nless \frac{D^3 \times T}{250} \times \frac{R}{W}$$

30. Losses in Pre-stress

I. Losses occuring in Post-tensioned members only :

(i) Loss due to friction

(ii) Loss due to slip of anchorage

II. Losses occuring in both Pre and Post-tensioned Members :

(i) Loss due to elastic shortening

(ii) Due to creep of concrete

(iii) Due to creep of steel

(iv) Due to shrinkage of concrete

31. The torsion wire is prestressed accurately to an extent equal to 100% of the scale reading. Then the sample is evenly distributed on the balance pan to counteract the prestressed torsion and the scale is brought back to the zero. As the sample dries, the loss in weight is continuously balanced by the rotation of a drum calibrated directly to read moisture % on wet basis.

32. Allen Hazen Equation

$$K = C.\, D_{10}^2$$

Where, D_{10} = Effective size in cm

K = Coefficient of permeability (cm)

$C = 100 - 150$

Note : It is valid only for particle size of soil 0.1 to 3.0 mm. So, It is valid for sand.

[PART : C]

33.

$$K = \frac{q \cdot l}{A \times h}$$

$$K = \frac{626 \times 18}{44.18 \times 60 \times 24.7}$$

$$= 1.72 \times 10^{-1} \text{ cm/sec}$$

$$A = \frac{\pi (7.5)^2}{4} = 44.18 \text{ cm}^2$$

Discharge-Velocity :

$$V = K.i = 1.72 \times 10^{-1} \times \frac{24.7}{18}$$

$$V = 2.36 \times 10^{-1} \text{ cm/sec}$$

Seepage Velocity :

$$V_s = \frac{V}{n} = \frac{2.36 \times 10^{-1}}{0.44} = 5.36 \times 10^{-1} \text{ cm/sec}$$

For $n_1 = 44\%$

$$e_1 = 0.79; \quad \frac{e_1^3}{1+e_1}$$

$$= \frac{0.79^3}{1+0.79} = 0.275$$

For $n_2 = 39\%$

$$e_2 = \frac{n_2}{1-n_2} = 0.64$$

So, $$\frac{e_2^3}{1+e_2} = \frac{0.64^3}{1+0.64} = 0.16$$

At 25°C, viscosity of water $\mu_1 = 8.95$ mili-poise at 20°C, $\mu_2 = 10.09$ mili poise :

Considering that,

$$K_1 : K_2 = \frac{\gamma_{w_1}}{\mu_1} : \frac{\gamma_{w_2}}{\mu_2}$$

or $$K_1 : K_2 = \frac{1}{\mu_1} : \frac{1}{\mu_2}$$

(neglecting effec on γ_w)

$$K_{20°C} = \frac{1.72 \times 10^{-1} \text{ cm/sec} \times 8.95}{10.09} = 1.526 \times 10^{-1} \text{ cm/sec}$$

at $e = 0.79$

Now considering that,

$$K_1 : K_2 = \frac{e_1^3}{1+e_1} : \frac{e_2^3}{1+e_2}$$

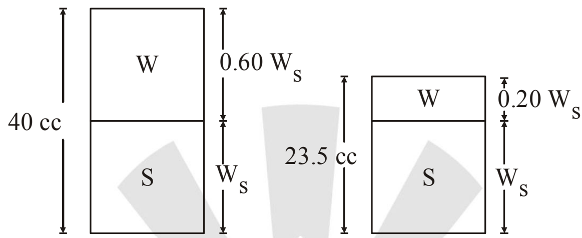
$K_{20°C}$, $e = 0.64$ is equal to

$$1.526 \times 10^{-1} \times 10 \times \frac{0.16}{0.275}$$

$$K_{20°C} = 0.8878 \times 10^{-1}$$

$$K_{20°C} = 8.88 \times 10^{-2} \text{ cm/sec}$$

34. The phase diagrams for the soil at liquid-limit and the shrinkage limit are shown in figure below:



Let the weight of the solids W_S,

Since the soil continuous to remain saturated till it reaches the state of consistency at shrinkage limit, the decrease in the volume of soil mass will be equal to the decrease in the volume of water. So,

$$\frac{0.60\, W_s - 0.20\, W_s}{\gamma_w} = \text{Volume at liquid limit} - \text{Volume at shrinkage limit}$$

i.e. $$\frac{0.40\, W_s}{\gamma_w} = (40 - 23.5)$$

$$\Rightarrow \quad 0.40\, W_S = 16.5 \times 1 \text{ gm/cc}$$

$$W_S = 41.25 \text{ gm}$$

Now, at shrinkage-limit

$$W_W = 0.20 \times 41.25 = 8.25 \text{ gm}$$

Volume of water $= V_W = \dfrac{8.25}{1} = 8.25$ cc and

$$V_S = 23.5 - 8.25 = 15.25 \text{ cc}$$

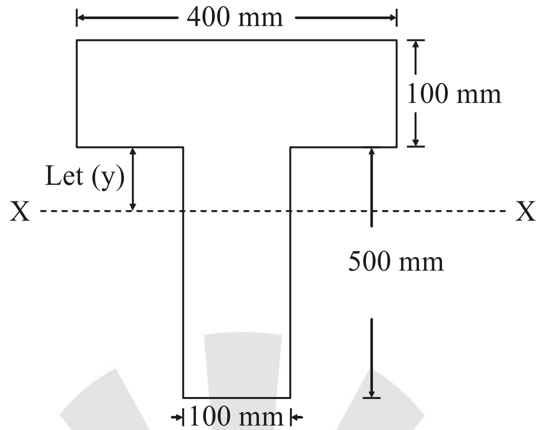
So, Specific gravity of solids.

$$G_S = \frac{W_s}{V_s \cdot \gamma_w}$$

$$= \frac{41.25}{15.25 \times 1} = 2.70$$

Shrinkage - Ratio, SR $= \dfrac{\gamma_d}{\gamma_w} = \dfrac{41.25}{23.5 \times 1} = 1.755$

35.



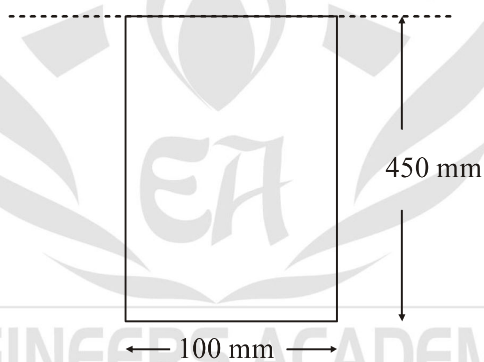
Let X-X is the equal area-axis, which is at 'y' distance from the flange.

For equal area axis :

$\Rightarrow (400\times100) + (y \times 100) = (500 - y) \times 100$

$\Rightarrow 100\ y + 100\ y = 50000 - 40000$

$$\Rightarrow \qquad y = \frac{10000}{200} = 50\ \text{mm}$$



Plastic moment capacity

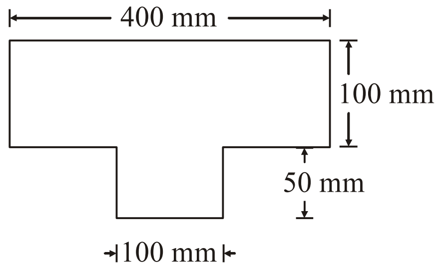
$$M_P = f_y.Z_P$$

Z_P = Plastic section modulus

$$Z_P = \frac{A}{Z}(\bar{y}_1 + \bar{y}_2)$$

$$\frac{A}{Z} = 100 \times 450 = 45000\ \text{mm}^2$$

$$\bar{y}_2 = \frac{450}{2} = 225\ \text{mm}$$



$$y_1 = \frac{(400\times100\times100)+(100\times50\times25)}{(400\times100)+(50\times100)}$$

$$y_1 = 91.67 \text{ mm}$$

So, $M_P = 250 \times (45{,}000)(225 + 91.67)$ N-mm

So, $M_P = 3562.5$ kN-m

36. Let the diameter of solid shaft be d_s. Let the external and internal diameter of the hollow shaft be d_o and d_i respectively.

It is given that both the shafts are made of same material, have same weight and length and are subjected to equal torsional force.

$\therefore$ Weight of solid shaft = Weight of hollow shaft

$$\Rightarrow \quad \gamma\frac{\pi}{4}d_s^2L = \gamma\frac{\pi}{4}(d_0^2 - d_i^2)L$$

$$d_s^2 = d_o^2 - d_i^2$$

Now, we know that torsional stiffness is given as

$$K = \frac{T}{\theta}$$

$$K = \frac{GI_P}{L}$$

For same value of G and L

$$K = \frac{T}{\theta} \propto I_P$$

$$\frac{K_{hollow}}{K_{solid}} = \frac{K_h}{K_s} = \frac{\frac{\pi}{32}(d_0^4 - d_i^4)}{\frac{\pi}{32}(d_s^4)}$$

$$\Rightarrow \quad \frac{K_h}{K_s} = \frac{(d_0^2 + d_i^2)(d_0^2 - d_i^2)}{(d_s^2)^2}$$

$$\Rightarrow \quad \frac{K_h}{K_s} = \frac{(d_0^2 + d_i^2)(d_o^2 - d_i^2)}{(d_o^2 - d_i^2)(d_o^2 - d_i^2)}$$

$$\Rightarrow \qquad \frac{K_h}{K_s} = \frac{d_o^2 + d_i^2}{d_o^2 - d_i^2}$$

Now, the quantity $\frac{d_o^2 + d_i^2}{d_o^2 - d_i^2}$ is always greater than '1'.

$\therefore \qquad K_h > K_S$

So, Torsional stiffness of hollow shaft will be more than solid shaft of equal weight and material.

37. ***Slope deflection equations:***

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right) \qquad \text{...(i)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right) \qquad \text{...(ii)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\delta}{L}\right) \text{...(iii)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right) \text{...(iv)}$$

Fixed end moments:

$$M_{FAB} = -\frac{Wab^2}{L^2} = -\frac{25 \times 3 \times 2^2}{5^2} = -12 \text{ kN-m}$$

$$M_{FBA} = +\frac{Wba^2}{L^2} = +\frac{25 \times 2 \times 3^2}{5^2} = +18 \text{ kN-m}$$

$$M_{FBC} = -\frac{Wl^2}{12} = -\frac{12 \times 8^2}{12} = -64 \text{ kN-m}$$

$$M_{FCB} = +\frac{Wl^2}{12} = +\frac{12 \times 8^2}{12} = +64 \text{ kN-m}$$

For fixed end $\theta_A = 0$, $\theta_C = 0$, $\theta_B \neq 0$, $\delta \neq 0$

Putting the values in (i), (ii), (iii) and (iv)

$$M_{AB} = -12 + \frac{2EI}{5}\left(2 \times 0 + \theta_B - \frac{3 \times (5 \times 10^{-3})}{5}\right)$$

$$M_{AB} = -12 + 0.4 \text{ EI}\theta_B - 0.0012 \text{ EI}$$

$M_{AB} = -12 + 0.4\ EI\theta_B - 108000$

$M_{AB} = 0.4\ EI\theta_B - 108012$

$$M_{BA} = +18 + \frac{2EI}{5}\left(2\theta_B + 0 - \frac{3\times(5\times10^{-3})}{5}\right)$$

$M_{BA} = 18 + 0.4\ EI\left(2\theta_B - 0.003\right)$

$M_{BA} = 18 + 0.8\ EI\theta_B - 108000$

$M_{BA} = 0.8\ EI\theta_B - 107982$

$$M_{BC} = -64 + \frac{2EI}{8}\left[2\theta_B + 0 - \frac{3\times(-5\times10^{-3})}{5}\right]$$

$$= -64 + \frac{2EI}{8}\left[2\theta_B + 0.003\right]$$

$M_{BC} = -64 + 0.5\ EI\theta_B + 67500$

$M_{BC} = 67436 + 0.5\ EI\theta_B$

$$M_{CB} = 64 + \frac{2EI}{8}\left[2\theta_C + \theta_B - \frac{3\times(-5\times10^{-3})}{5}\right]$$

$M_{CB} = 64 + 0.25\ EI\theta_B + 67500$

$M_{CB} = 0.25\ EI\theta_B + 67564$

For equilibrium of Joint 'B'

$M_{BC} + M_{BA} = 0$

$\Rightarrow 0.8\ EI\theta_B - 107982 + 67436 + 0.5\ EI\theta_B = 0$

$\Rightarrow \quad 1.3\ EI\theta_B = 40546$

$\Rightarrow \quad EI\theta_B = 31189.23$

Putting the value of $EI\theta_B$ in all equations

$M_{AB} = (0.4 \times 31189.23) - 108012 = -95536.3$ N-m

$M_{AB} = -95.54$ kN-m

$M_{BA} = (0.8 \times 31189.23) - 107982$

$= -83030.62$ N-m

$M_{BA} = -83.03$ kN-m

$M_{BC} = 67436 + (0.5 \times 31189.23)$

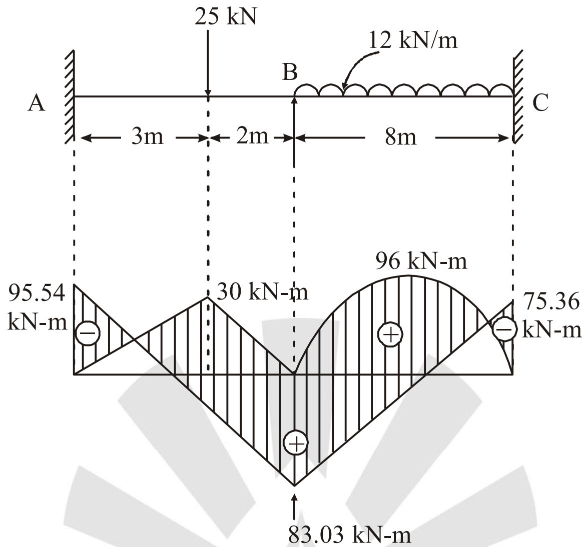
$= 83030.62$ N-m

$M_{BC} = +83.03$ kN-m

$M_{CB} = (0.25 \times 31189.23) + 67564$

$= 75361.3$ N-m

$$M_{CB} = 75.36 \text{ kN-m}$$



38. Since the beam is an Isolated T-beam :

$$l_0 = 0.7l = 0.7 \times 8$$
$$= 5.6 \text{ m (due to fixed ends)}$$

effective-width of flange of T-beam :

$$B_F = \frac{l_0}{\left(\frac{l_0}{B}\right)+4} + b_w = \frac{5600}{\left(\frac{5600}{1000}\right)+4} + 300 = 883.33\text{mm}$$

$$b_w = 300 \text{ mm}$$

effective depth (d) = 600 mm

$$D_F = 120 \text{ mm}$$

$$A_{ST} = 5 \times \frac{\pi}{4} \times 25^2 = 2454.375 \text{ mm}^2$$

For moment of resistance

(i) Let $x_u < D_F$

i.e. Neutral Axis lies within the flange.

For x_u $C = T$

$$0.36 f_{ck} B_F x_u = 0.87 f_y A_{ST}$$

$$x_u = \frac{0.87 \times 2454.375 \times 415}{0.36 \times 20 \times 883.33}$$

$$x_u = 139.40 \text{ mm} > D_F$$

So, Assumption (i) is wrong.

(ii) Let $\frac{3}{7}x_u \geq D_F$

For x_u $\quad C = T$

$\Rightarrow 0.36f_{ck}b_W x_u + 0.45f_{ck}(B_F - b_w)D_F = 0.87f_y A_{st}$

$\Rightarrow (0.36\times20\times300\ x_u) + 0.45\times20(833.33-300)120 = 0.87\times415\times 2454.375$

$\Rightarrow \quad x_u = 138.59$ mm

$$\frac{3}{7}x_u = 59.157 < D_F$$

So, the (ii) assumption is also wrong.

(iii) $\quad x_u > D_F$

$$\frac{3}{7}x_u < D_F$$

So, For x_u $\quad C = T$

$\Rightarrow 0.36f_{ck}b_w x_u + 0.45f_{ck}(B_F - b_W)y_f = 0.87f_y A_{ST}$

$\Rightarrow 0.36\times20\times300x_u + 0.45\times20(883.33 - 300)$

$$\{(0.15x_4 + 0.65\times120)\} = 0.87\times415\times5\times\frac{\pi}{4}(25)^2$$

$\Rightarrow 2160x_u + (787.49x_u + 409497.66) = 886152.09$

$$\Rightarrow \quad x_u = \frac{476654.43}{2947.49} = 161.71 \text{ mm}$$

So, $\quad x_u > D_F$

$$\frac{3}{7}x_u = 69.30 < D_F$$

Now, $\quad x_{u_{lim}} = 0.48$ d

$$x_{u_{lim}} = 0.48 \times 600 = 288 \text{ mm}$$

$x_u < x_{u_{lim}}$ $\quad$ under R/F section.

$$\therefore \text{M.R.} = 0.36\ f_{ck}b_w x_u(d - 0.42\ x_u) + 0.45f_{ck}(B_F - b_W)y_F\left(d - \frac{y_F}{2}\right)$$

M.R. $\Rightarrow \quad y_F = 0.15\ x_u + 0.65\ D_F$

$$= (0.15 \times 161.71) + (0.65 \times 120) = 102.25 \text{ mm}$$

$$\text{MR} = (0.36\times20\times300\times161.71) + 0.45\times20(883.33-300)\times102.25 \times\left(600 - \frac{102.25}{2}\right)$$

$$MR = 349293.6 + 294641277.3$$
$$MR = 294.99 \times 10^6 \text{ N-mm}$$
$$MR = 294.99 \text{ kN-m}$$

39. (i) Bearing stresses at Bends :

The bearing stress inside a bend is any other bend shall be calculated as given below :

$$\text{Bearing stress} = \frac{F_{BT}}{r \cdot \phi}$$

Where, F_{BT} = Tensile force due to design loads in a bar or group of bars

r = internal radius of the bend

ϕ = size of the bar or, in bundle, the size of bar of equivalent area

for limit state method of design this stress shall not exceed $\dfrac{1.5 f_{ck}}{1+\left(\dfrac{2\phi}{a}\right)}$.

where, $f_{ck} \rightarrow$ characteristic strength

$\phi \rightarrow$ as given above

$a \rightarrow$ center to center distance between bars or group of bars perpendicular of the plan of bend.

(ii) Conditions for curtailment of flexural R/F in tension:

(a) The shear at the cutoff point doesnot exceed two-third that permitted, including the shear strength of web reinforcement provided.

(b) Stirrup area in excess of that required for shear and torsion is provided along each terminated bar over a distance from the cutoff point equal of three-fourths. The effective depth of the member excess stirrup area shall be not less than 0.4 $b.S/f_y$, where 'b' is the breadth of the beam, S is the spacing and f_y is the characteristic strength of reinforcement in N/mm^2. The resulting spacing shall not exceed "$d/8\ \beta_b$". Where β_b is the ratio of the area of bars cutoff to the total area of bars at the section, and 'd' is the effective depth.

(c) For 36 mm and smaller bars, the continuing bars provide double the area required for flexure at the cutoff point and the shear doesn't exceed three-fourth that permitted.

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